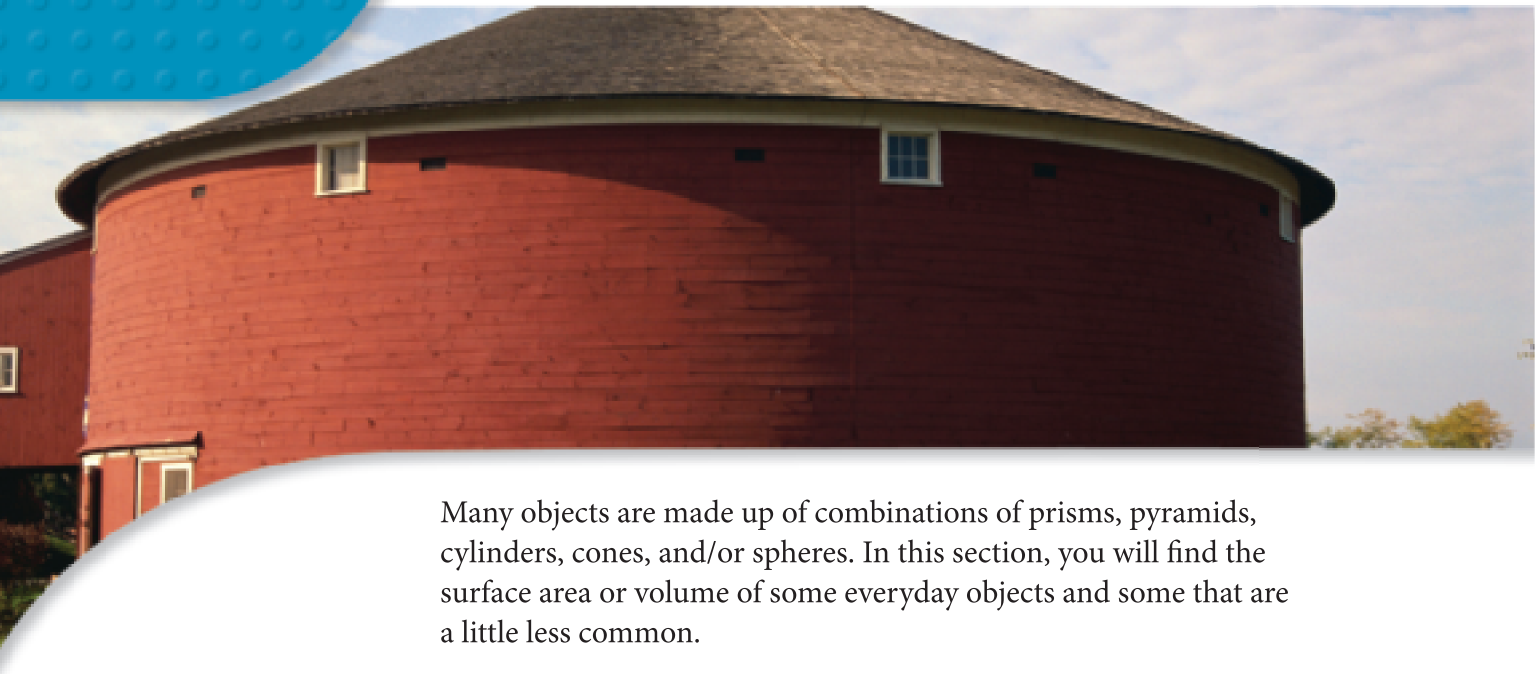
9.5 Solve Problems Involving Surface Area and Volume

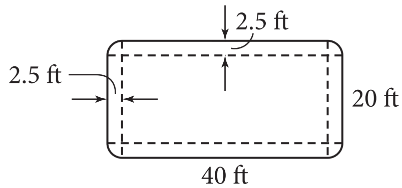


Many objects are made up of combinations of prisms, pyramids, cylinders, cones, and/or spheres. In this section, you will find the surface area or volume of some everyday objects and some that are a little less common.

Investigate

Surface Area and Volume of a Composite Solid

To make a backyard hockey rink package, a company has developed a plastic bladder that can be filled with water. Once the water freezes, the top is removed to expose the ice. The bladder is made of high-strength plastic, and is filled with water to a depth of 4 in. The top and bottom surfaces are rectangular with rounded corners. The total length of the rink is 40 ft and the total width is 20 ft.



a) Describe the figures that form the surface of the ice. Find the area of the ice surface.

b) Which combination of objects make up the bladder? Find the surface area of plastic used in the bladder.

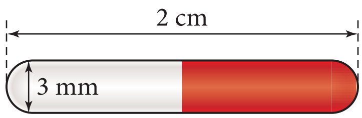
c) One cubic foot is equivalent to 7.48 gal. Suppose water pours out of a garden hose at a rate of 3 gal/min. How long will it take to fill the bladder?

d) The cost of the plastic material is 27¢/ft^2. Find the total cost of manufacturing the bladder.

Example 1

Volume of a Pill Capsule

A pill capsule is in the shape of a cylinder with half of a sphere (a hemisphere) on each end. The length of the cylindrical portion is 2 cm and the diameter is 3 mm. Find the volume of the capsule.



Solution

The pill capsule is a cylinder with two hemispheres, which are equivalent to a sphere.

$$V = V_{cylinder} + V_{sphere}$$
$$= \pi r^2 h + \frac{4}{3}\pi r^3$$
$$= \pi(0.15)^2(2) + \frac{4}{3}\pi(0.15)^3$$

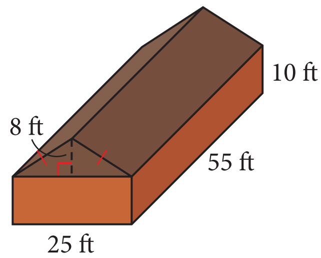
Since d = 0.3 cm, r = 0.15 cm.

$$\doteq 0.156$$

The volume of the pill capsule is approximately 0.16 cm^3.

Example 2 Surface Area of a Barn

The exterior walls of a barn are to be painted. The barn is in the shape of a rectangular prism with an isosceles triangular prism for a roof.



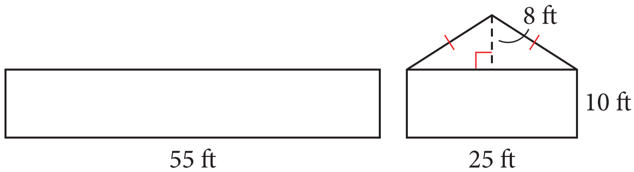
a) Find the total area to be painted.

b) The paint is sold in 1 gal cans. On the first coat of paint, a gallon of paint will cover an area of 400 ft^2. How many gallons of paint are needed for the first coat?

c) On the second coat of paint, a gallon will cover an area of 525 ft^2. How many gallons of paint are needed for both coats?

Solution

a) Sketch the walls that are to be painted.



Find the total area of the rectangular parts of the walls.

$$A = 2(55 \times 10) + 2(25 \times 10)$$
$$= 1600$$

Find the area of the triangular parts of the walls.

$$A = 2 \times \left(\frac{1}{2}\right)(25)(8)$$
$$= 200$$

Find the total area to be painted.

$$A = 1600 + 200$$
$$= 1800$$

An area of 1800 sq ft is to be painted.

b) Divide the total area by the area covered by each gallon of paint.

$$= \frac{1800}{400}$$
$$= 4.5$$

The barn will need 4.5 gal of paint for the first coat.

c) Find the amount of paint needed for the second coat.

$= \frac{1800}{525}$

$= 3.4$

Find the total amount of paint needed.

$= 4.5 + 3.4$

$= 7.9$

Five gallons of paint need to be purchased for the first coat, but there will be a half-gallon left over. Since 3.4 gallons are needed for the second coat, only 3 additional gallons need to be purchased.

Eight gallons of paint are needed.

Example 3 Combination of a Rectangular Prism and a Cylinder

A piece of wood is 8 in. wide, 2 ft long, and 1 in. high. Twelve holes, each with diameter 2 in. are drilled through the wood.

a) Find the volume of the piece of wood before the holes are drilled.

b) How much material is removed by the drill?

Solution

a) Find the volume of the piece of wood.

$V = l \times w \times h$

$= (24)(8)(1)$

$= 192$

Two feet is 24 in.

The original volume of wood is 192 in^3.

b) Find the amount of wood removed for each hole.

Each hole is a cylinder with radius 1 in. and height 1 in.

For one hole,

$V = \text{area of base} \times \text{height}$

$= \pi r^2 \times h$

$= \pi(1)^2(1)$

$\doteq 3.14$

Find the total amount of wood removed.

12×3.14

$= 37.68$

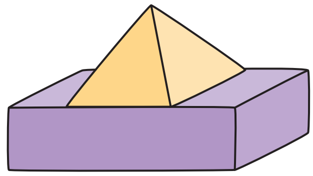
The total amount of wood removed is approximately 37.68 in^3.

Key Concepts

- When a figure is made up of a combination of shapes, use the appropriate formula for each shape to find the total required quantity.
- It is important to read questions carefully and to plan the steps of your solution.

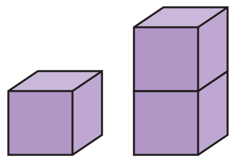
Discuss the Concepts

D1. An inflatable toy has the shape of a small square-based pyramid on top of a rectangular prism. Suppose you are asked to find the surface area of the toy. Explain why you cannot add the surface area of the pyramid to the surface area of the prism.



D2. To find the total volume of an object made up of more than one three-dimensional shape, Darnell found the volume of the individual shapes and added them together. Is Darnell correct? Explain your answer.

D3. The surface area of a cube with sides 2 cm long is 24 cm^2. The total surface area of two such cubes standing alone is 48 cm^2. But, the two cubes are stacked, the total exposed surface area is (48 — 8) or 40 cm^2. Extend this pattern to find the total exposed surface area for three cubes and for four cubes stacked.



Practise the Concepts A

Math Connect

Silage is made by chopping and fermenting corn, alfalfa, or other grass or grain crops. It is stored in storage silos and fed to dairy cattle and sheep.

For help with questions 1 and 2, refer to Example 1.

1. A storage silo is in the shape of a cylinder with a hemisphere at the top. The total height of the silo is 35 ft. The circumference of the cylinder is 22 ft.

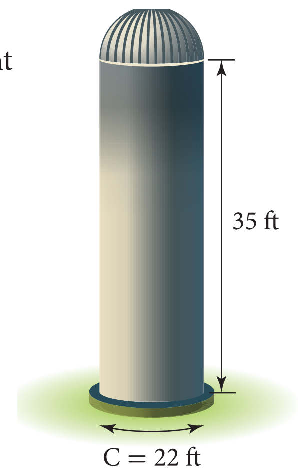
a) Find the radius of the silo.

b) Find the height of the cylindrical portion of the silo.

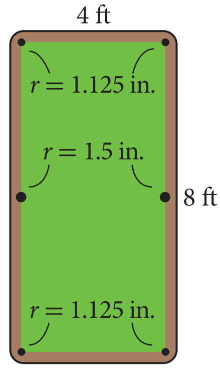
c) Find the volume of the cylindrical portion of the silo.

d) Find the volume of the hemispherical portion of the silo.

e) What is the total volume of the silo?



2. The slate of a rectangular pool table has a width of 4 ft, a length of 8 ft, and a thickness of 2 in. Pockets are cut as shown in the diagram.



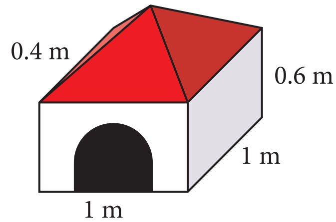
a) Find the volume of slate removed for each pocket.

b) Find the volume of the slate in cubic feet.

c) If 1 ft^3 of slate weighs 166.6 lbs, what is the weight of the slate?

For help with question 3, refer to Example 2.

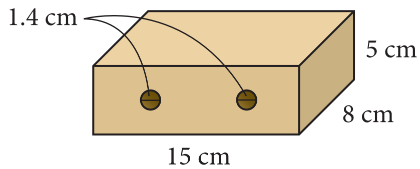
3. A doghouse in the shape of a square-based prism has a roof in the shape of a square-based pyramid. Find the total surface area that needs to be painted. Subtract 0.2 m^2 for the cut out doorway.



Apply the Concepts B

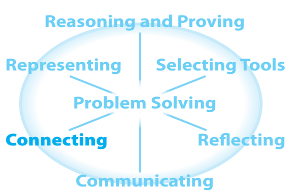
For help with question 4, refer to Example 3.

4. Ben is making a child's toy car from a rectangular block of wood. He drills two holes, each with diameter 1.4 cm, through the block for axles to support the wheels. The block of wood has length 15 cm, width 8 cm, and height 5 cm.



a) What is the volume of the block of wood before the holes are drilled?

b) What volume of wood remains after the holes are drilled?



5. Plans for a new theatre call for a hemispherical dome to be placed on top of a cube-shaped theatre. The side lengths of the cube are equal to the diameter of the dome. To adequately supply fresh air to the building, the engineers need to know the volume of air in the theatre. The radius of the dome is 155 ft.

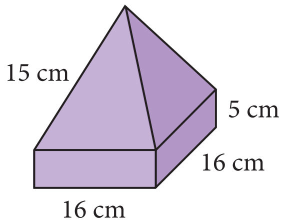
a) Find the volume of the cube-shaped portion of the theatre.

b) Find the volume of the hemispherical portion of the theatre.

c) Find the total volume of air in the theatre.

Chapter Problem

6. Vanessa has decided to package two items together for the holidays. She plans to market a combination of the toque in a pyramid attached on top of a square-based prism that will hold a pair of ski gloves.

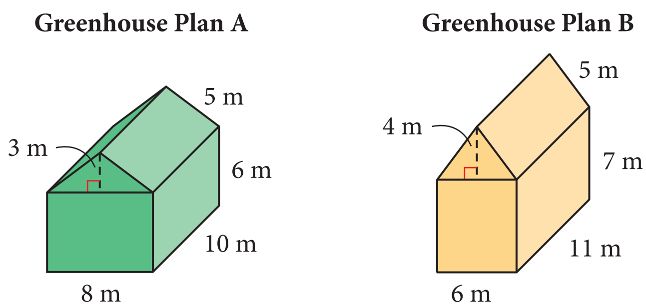


a) Find the surface area of this package.

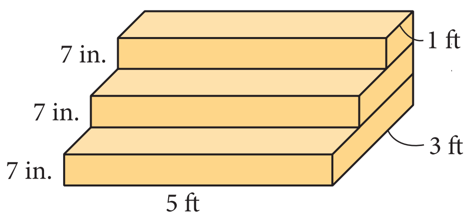
b) What is the volume of the package? The height of the pyramid is 10.2 cm.

Achievement Check

7. Arthur is comparing two greenhouse designs, shown below. To allow for ventilation and irrigation systems, Arthur should choose the design that has more space in the peaked roof area. Which greenhouse design should Arthur choose? Why?



8. This set of stairs is positioned on a garage floor, against a wall.

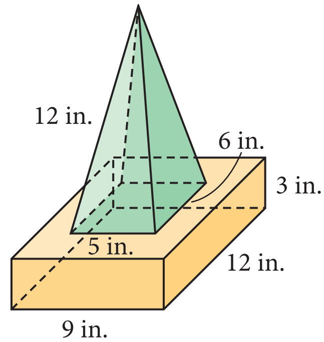


a) The stairs are to be painted. Find the area that needs to be painted.

b) If 1 L of paint covers 11.3 ft^2, how many litres of paint are needed to paint the stairs?

9. A trophy shop has a design for a plaque that needs to be silver-plated.

If the bottom side of the prism and the contact surface of the pyramid and the prism are the only surfaces that do not need to be silver-plated, find the surface area that needs to be silver-plated.



Literacy Connect

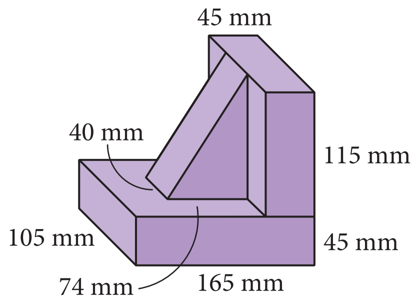
10. A variety of materials and shapes are used in designing all structures.

a) List some of the shapes commonly used in building homes.

b) Give some reasons why architects design buildings and homes to include a variety of shapes.

Extend the Concepts C

11. Find the volume of this object.



12. A designer is making a scale model of a garbage can to take to different manufacturers to get estimates of the cost to manufacture 10 000 units.

The shape of the garbage can is a cylinder with a hemisphere on the top. If the cylindrical portion is 1 m tall with radius 65 cm, and the designer has made a scale model of the object where she reduced all of the measurements by 40%, find

a) the volume of the actual garbage can.

b) the volume of the scale model.

c) the percent decrease in the volume of the scale model compared to the actual object.